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A TOPOLOGICAL ANALYSIS OF ELECTRONIC CIRCUITS BY A PARAMETER EXTRACTION METHOD*

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The problem of deriving an expression for symbolic network functions is considered. A network approach to this problem, proposed by Feussner, is extended to networks with controlled sources. The method proposed here, unlike that proposed by Barrows and Hoang, does not require laborious enumeration of transfer loop circuits.

Key words: symbolic network function, impedance, admittance, controlled source, ideal operational amplifier, extraction of a branch.

ONE HUNDRED and fifty years ago Kirchhoff formulated the problem of making a direct analysis of an electric circuit without deriving the equations of electrical equlibrium [1]. Here, the problem of analysing an electric circuit is reduced to finding the symbolic network functions (SNF) in the form

$$H = \Delta_N / \Delta_D , \tag{1}$$

where Δ_N and Δ_D are the numerator and denominator of the SNF respectively, in which the parameters of all the circuit components are represented by symbols [2].

Feussner must be considered the founder of the circuit approach to obtaining SNF. He turned his attention to the difficulties of constructing SNF using Kirchhoff's and Maxwell's topological formulae [3, 4]. Feussner reduced the derivation of SNF to an expansion of the determinants of the initial network and of the networks derived from it. The network determinant is expanded using one of two formulae, depending on the type of network components:

$$\Delta = y\,\Delta_y + \Delta^y \tag{2}$$

and

$$\Delta = z\,\Delta^z + \Delta_z , \tag{3}$$

where the subscript on the symbol Δ indicates contraction of the y- or z-branch, and the superscript indicates their removal.

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If the initial or derived network can be decomposed into two subnetworks at nodes a or b then the diakoptic formula of bisection is used to expand the network determinant

$$\Delta = \Delta_1 (a,b)\, \Delta_2 + \Delta_1\, \Delta_2 (a,b)\,. \tag{4}$$

where Δ_1 and Δ_2 are the determinants of the first and second subnetworks. The notation in parenthesis after Δ indicates the merging of the external nodes in the corresponding subnetworks.

Recursive use of formulae (2)—(4) enables the network determinant to be represented immediately in its final form, i.e. in a compact form with the common multipliers taken outside the parethesis. Meanwhile, expanded symbolic expressions, which are composed of tree weights and the complements of network trees from Maxwell and Kirchhoff need additional laborious transformations.

In order to determine the SNF numerator Feussner introduced the concept of the transfer loop of a network, which necessarily contains an independent source and a branch with the required response. Here, the SNF numerator is expanded using the formula

$$\Delta_N = \sum_{i \in \rho} P_i \Delta_i\,, \tag{5}$$

where ρ is the set of transfer loops of the network, P_i is the product of the admittances which occur in the ith transfer loop and Δ_i is the determinant of the network, which is formed from the initial network by contracting all branches of the ith transfer loop. The denominator of the SNF is equal to the determinant of the derived network, constructed as a result of contraction (elimination) of the independent voltage (current) source and the branch with the required current (voltage).

Formulae (2)—(5) provide the simplest solution of the problem of symbolic analysis of passive electric networks containing no mutual inductances, and these formulae have been used in the textbook [5]. Attempts have been made by Barrows [6] and Hoang [7] to derive a similar solution for networks containing mutual inductances and controlled sources. They used the concept of a chain of transfer loops of the network with a controlled source, which generalized the concept of a transfer loop and a corresponding extension of formula (5) to obtain both the numerator and denominator of the SNF. However, because of the laborious enumeration of the transfer loop chains these methods are difficult to use and are not sufficiently effective for analysing networks with several controlled sources.

The development of a network approach based on an extension of formulae (2) and (3) to extract the parameters of the controlled source [8] is more promising. It enables one to develop a standard procedure for determining the numerator and denominator, without using the concept of a chain of transfer loops. It is well known that the numerator of the transfer SNF can be found by considering the network developed from the initial network by transforming the independent source into a non-eliminated controlled source (NECS) which is controlled by the required response [6, 7]. The parameter of the NECS occurs in all terms of the determinant of such a network. If this parameter is given a value of units the numerator of the SNF will be equal to the determinant of the corresponding network.

Hence, the determination of the SNF is ensured by expanding the determinants of the network models of the numerator and denominator without using formula (5) or its extensions, A method of extracting the parameters for expanding the network determinant is proposed below which, unlike the topological method of extracting the branches and arcs [9] and the matrix method of parameter extraction [10], does not require intermediate mathematical models in the form of graphs or matrices. In this case mutually cancelled terms, i.e. duplications, do not occur, these duplications being caused both by redundancy of the intermediate models and by the absence of a common node in the independent source and in the branch with the response required. Recall, that the second type duplicaion arises when the numerator of the transfer SNF is determined in the form of the difference between the cofactors of the matrix or graph [5].

The topological method of isolating the parameters. Suppose the network contains passive components and controlled sources of all four types: 1) a voltage controlled current source (VCCS), 2) a current controlled voltage source (CCVS), 3) a voltage controlled voltage source (VCVS) and 4) a current controlled current source (CCCS). For brevity, we shall call the controlled branches of the controlled sources generators, and the controlling branches of the controlled sources receivers.

The network approach does not require one to choose one of the network nodes as a basis node [6, 7]. Consequently, we can begin to expand the network determinant by isolating the parameters of the passive components from formulae (2) and (3). It is appropriate, first, to combine the parallel-connected y-branches and the series-connected z-branches. The parallel-(series) connected VCCS (CCVS) are also replaced by a single controlled source, the parameter of which is equal to the sum of the parameters of the controlled sources which formed them, taking the signs into account.

After the operation of contraction has been performed it is necessary to consider the possibility of simplifying the derived network by removing y-loops and y-branches which are connected in parallel with the voltage generators or the current receivers. In view of duality, the simplifications mentioned correspond to contraction of suspended z-branches which are connected in series with the current generators or voltage receivers. These elemenary simplifications do not change the value of the network determinant, which is obvious from physical considerations or follows from formulae (2) and (3).

Extraction of the passive components may lead to degeneration of the controlled source in derived networks. It is an important feature and advantage of the method proposed. A voltage controlled current circuit is said to be degenerate when its generator and receiver are connected in parallel. Such a VCCS is

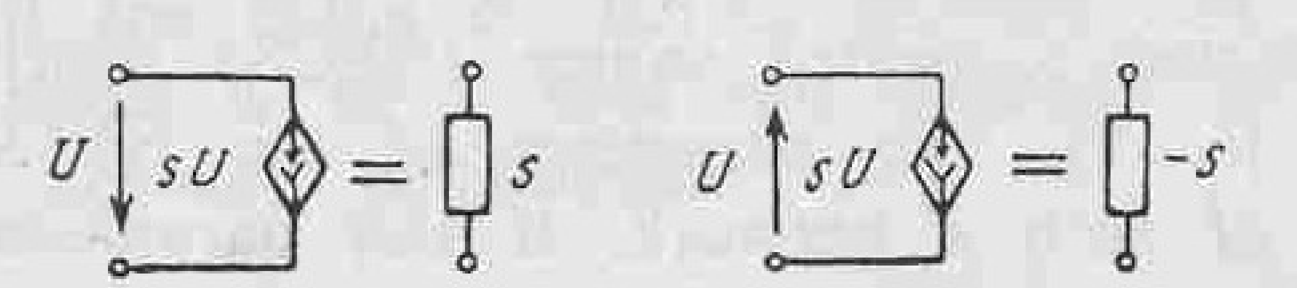

FIG. 1

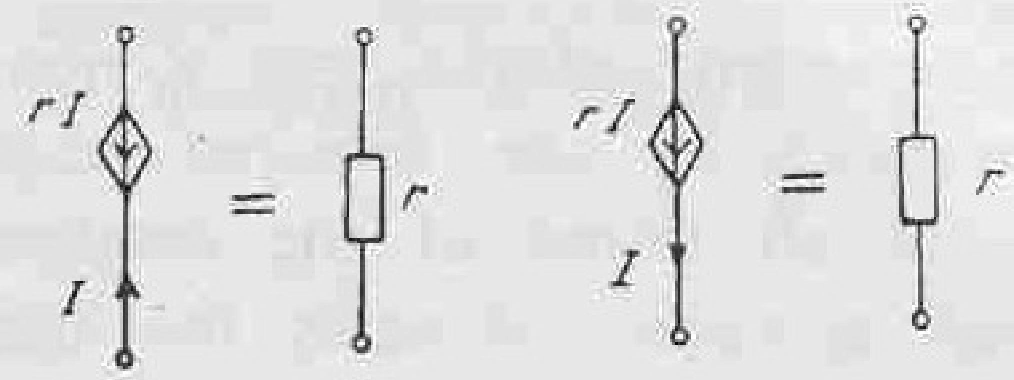

FIG. 2

replaced by a quasipassive two-terminal network with an admittance parameter in accordance with Fig. 1.

In view of duality, a CCVS is considered to be degenerate if its generator and receiver are connected in series. A degenerate CCVS is transformed into a quasipassive two-terminal network with a resistance parameter, as shown in Fig. 2.

It is notable that quasipassive two-terminal networks have the status of passive y- or z-branches. They can therefore be combined with ordinary y- or z- branches and be extracted taking the sign into account using formulae (2) or (3).

Recursive application of formulae (2) and (3) in conjunction with quasipassive transformations can produce derived network in which there are no passive components. Furthermore, immediate extraction of the parameters of the controlled sources is necessary to provide the sensitivity functions to the parameters of the active components [9]. For these cases a formula for extracting the parameter of the controlled source is proposed which generalizes formulae (2) and (3), and also a formula for extracting the parameter of the VCCS [8].

Suppose χ is a generalized parameter, by which we mean a parameter of a controlled source of one type or another. Then, the formula for extracting the parameter χ from the structure will not differ from formula (2):

$$\Delta = \chi \Delta_{\chi} + \Delta^{\chi}, \tag{6}$$

where Δ_{χ} is a determinant of the first derivative of the network, developed from the initial network by assigning to the extracted controlled source the status of the NECS with parameter equal to unity and Δ^{χ} is a determinant of the second derivative of the network, which is formed by eliminating the extracted controlled source from the initial network.

Recall, that elimination of the VCCS reduces to removing its generator and receiver from the network, which, unlike the elimination of controlled sources of other types, does not reduce the number of nodes in the network. The removal of a controlled voltage source consists of contracting its generator and receiver. If a VCVS (CCCS) is removed a generator (receiver) branch is contracted and a receiver (generator) branch is removed.

As can be seen, the specific features of any controlled source are reflected in the second term of formula (6). The assignment to the extracted controlled source of the status of NECS will prevent repeated use of this formula for the same controlled circuit. Thus, NECS are not distinguished by their physical properties, and a controlled source of some type which has been generated by some NECS is of no importance. The proof of formula (6) follows immediately from the generalized unistor graph method [11] and the above rule for finding the numerator of the SNF by transforming the independent source into a NECS.

In many practical cases a network may contain a controlled source with parameters having an infinitely large value, for example, operational voltage amplifiers. The use of the NECS concept simplifies the derivation of the SNF without requiring the preliminary construction of a common symbolic expression followed by its laborious transformation. In particular, operational amplifiers are replaced by NECS. The parameters of the operational amplifiers are factors both in the numerator, and in the denominator of SNF. Consequently, by assigning to the parameters of the NECS values of unity or minus unity it is possible to prevent need for their subsequent contraction.

By isolating the parameters of the passive components and controlled sources, the analysis of an arbitrary active network can be reduced to the analysis of a number of the elementary active networks (EAN); an elementary active network is a network which contains exclusively NECS, the parameters of which are equal to unity. Before finding the determinant of the EAN it is necessary to test it for degeneracy. The network is degenerate if its determinant is identically equal to zero. An elementary active network can be regarded as a network with a VCCS, and represented by a current-voltage graph [12]. It follows from this that the generators necessarily form a tree of a non-degenerate EAN, and the receivers are contained in a complement of this tree, and vice versa.

By virtue of this feature, the current-voltage graph corresponding to the non-degenerate EAN will contain a single complete tree. Consequently, the value of the determinant of such an EAN is equal in absolute value to the product of the parameters of the NECSs which form this network, i.e. to unity. The procedure for finding the sign of the EAS from its structure is identical with Coates's procedure for calculating the sign of a complete tree of a current-voltage graph [12].

An easily recognizable feature of the degeneracy of a network with an NECS follows from the property of a non-degenerate EAN. The determinant of the network is identically equal to zero, if this network contains at least one loop or section composed exclusively of generators or exclusively of receivers of the NECS. It is useful to take this feature into account when considering loops and sections which contain controlled sources and an NECS. Four types of connections are possible with respect to the extracted controlled source in accordance with formula (6): 1) the controlled source can be transformed into the NECS and eliminated, 2) the controlled source cannot be transformed into the NECS and eliminated, 3) the controlled source cannot be transformed into the NECS but it may be eliminated and 4) the controlled source can be transformed into the NECS but cannot be eliminated. It is of interest to analyse the second, third and fourth cases.

In the second form of connection the network is degenerate. It occurs in the following cases: 1) the generator of at least one CCVS or VCVS forms a loop with the generators of the NECS, 2) the receiver of at least one CCVS or CCCS forms a loop with the receivers of the NECS, 3) the generator of at least one VCCS or CCCS forms a section with the generators of the NECS and 4) the receiver of at least one VCCS or VCVS forms a section with the receivers of the NECS.

Taking into account the third form of connection we can simplify the network before expanding its determinant by elimination: 1) a *y*-branch which forms a loop with the generators of the NECS or with the receivers of the NECS, 2) a VCCS or CCCS, the generator of which forms a loop with the generators of the NECS, 3) a VCCS or VCVS, the receiver of which forms a loop with the receivers of the NECS, 4) a *z*-branch which forms a section with the generators or receivers of the NECS, 5) a CCVS or VCVS, the generator of which forms a section with the generators of the NECS, and 6) a CCVS or CCCS, the receiver of which forms a section with the receivers of the NECS. It should be noted that the elimination of a *z*-branch here, as distinct from formula (3), means the elimination of a degenerate CCVS, essentially, is what such a branch is (Fig. 2).

Finally, for the fourth type of connection it is possible to simplify the network by transforming it into the NECS: 1) a *z*-branch which forms a loop with the generators or receivers of the NECS, 2) a CCVS or VCVS, the generator of which forms a loop with the receivers of the NECS, 3) a CCVS or CCCS, the receiver of which forms a loop with the generators of the NECS, 4) a *y*-branch which forms a section with the generators or receivers of the NECS, 5) a VCCS or CCCS, the generator of which forms a section with the receivers of the NECS, and 6) a VCCS or VCVS, the receiver of which forms a section with the generators of the NECS. Transformation into an NECS involves taking the parameter of the controlled source outside the brackets and assigning to this controlled source in the network the status of an NECS with parameters equal to unity. Transformation of *y*-branch (*z*-branch) into the NECS means taking its parameter outside the brackets and the contraction (elimination) of this branch from the network.

To avoid unnecessary calculations it is best to investigate the network for degeneracy or the possibility of carrying out simplification before each extraction of the parameters of the passive components and controlled sources (until a large number of NECSs appear in the network). It is first necessary to check the connectivity of the network. It is then useful to consider the loops and sections formed by the controlled sources. A generalization of the above forms of connection is carried out using formula (6). To check a network for degeneracy the following features are used, which are easily recognizable from its figure: 1) there is at least one loop in the network, which is formed only by voltage generators or only by current receivers, and 2) there is at least one section in the network which is formed only by current generators of only by voltage receivers. A consequence of the first feature is that voltage generators and current receivers cannot form loops, otherwise the network is degenerate.

It should be noted that among voltage and current generators there can be generators of the NECS, and among voltage and current receivers there can be receivers of the NECS. This agrees with the types of connection of controlled sources and the NECS. The generalized feature of degeneracy of the network enable us to formulate rules for its simplification by means of elimination and by transformation into an NECS. A network is simplified as a result of elimination: 1) a y-branch which forms a loop with voltage generators or with current receivers, 2) a VCCS or CCCS, the generator of which forms a loop with voltage generators, 3) a VCCS or VCVS, the receiver of which forms a loop with current receivers, 4) a z-branch, which forms a section with current generators or with voltage receivers, 5) a CCVS or VCVS, the generator of which forms a section with current generators, and 6) a CCVS or CCCS, the receiver of which forms a section with voltage receivers.

A network can be simplified by transformation into the NECS: 1) a z-branch, which forms a loop with voltage generators or with current receivers, 2) a CCVS or VCVS, the generator of which forms a loop with current receivers, 3) a CCVS or CCCS, the receiver of which forms a loop with voltage generators, 4) a y-branch which forms a section with current generators or with voltage receivers, 5) a VCCS or CCCS, the generator of which forms a section with voltage receivers, and a VCCS or VCVS, the receiver of which forms a section with current generators.

Checking the network or the derived network for degeneracy in conjunction with network simplifications considerably reduces the amount of calculation when setting up the SNF, due to the reduction in the number of EAN to be considered. The above rules are governed by the duality principle and do not require formal memorization, since they are completely in accord with physical concepts on passive components and controlled sources. The operation of transformation into an NECS (see formula (6)) generalizes the operations of contraction of a y-branch and elimination of a z-branch in formulae (2) and (3). This establishes that the operation of contraction is nonexistent for controlled sources. Consequently, systematic extraction of the passive components and controlled sources can be carried out by replacing the operations of contraction of a y-branch (elimination of a z-branch) by the operation of transformation into an NECS, elimination of a z-branch being more correct from a physical point of view than elimination of the CCVS, which replaces it.

The weak point of the method proposed is having to use the procedure for finding the sign of the determinant of a non-degenerate EAN. Nevertheless, the topological method of extracting the parameter form derived above can be recommended as one of the most effective symbolic methods for analysing linear networks with controlled sources of all types, including ideal controlled sources. The well-known methods of constructing the SNF using matrix [10—13], graph [5, 9—11, 14—17] and set-theoretic [18] approaches are not sufficiently universal or clear. Their use involves a large number of duplications, which, are first found, and then eliminated with considerable difficulty.

The analysis of networks by parts and special cases of parameter extraction are considered below. The results obtained enable the network determinants to be expanded so as to minimize the number of times one needs to use a topological procedure to find the sign, or eliminate its use entirely

Determination of the parameters of subnetworks. When analysing complex networks, it is convenient to combine the extraction of the individual components using formulae (2), (3) and (6) and the extraction of subnetwork parameters using diakoptic formulae. Practical networks with controlled sources have, as a rule, a cascade structure, so the most useful formulae are the formulae of two- and three-node bisection. A formula for the bisection of passive network with respect to three nodes a, b and c was obtained by Galyamichev [14]:

$$\Delta = \Delta_1(a,b,c)\,\Delta_2 + \Delta_1\,\Delta_2(a,b,c) + \Delta_1(cb,a)\times$$

$$\times\Delta_2(b,c) + \Delta_1(ca,b)\,\Delta_2(a,c) + \Delta_1(ab,c)\,\Delta_2(a,b). \qquad (7)$$

In this formula the same notation is used as in formula (4). Here $\Delta_1(a,b,c)$ and $\Delta_2(a,b,c)$ are determinants derived from the first and second subnetwork by combining the external nodes. Notation of the form (ab,c), unlike the designated notation, does not indicate any transformations of the corresponding subnetwork. To find this factor it is necessary enumerate in the subnetwork considered all 2-trees which contain a path from node a to node b, without passing through node c [5].

We will now consider the fact that the weights of path 2-trees with a code (ab,c) are terms of the numerator of a transfer SNF [11]. Here, an independent source is connected between nodes a and c, the response being taken between nodes b and c. Hence, a factor of the form (ab,c) in formula (7) can be found as the determinant of the network which is formed from the corresponding subnetwork by connecting the NECS (ac,bc). The first pair of numbers in parenthesis indicates the nodes to which the generators are connected, and the second indicates the nodes where the receivers are connected. The parameter of the NECS is taken as equal to unity, as when finding the numerator of the SNF.

Hence, all factors, both for formula (7), and for formula (4), can be obtained in terms of network determinants. The connection of the NECS enables one to represent the external characteristics of the subnetwork in the form of derived networks, thereby avoiding the use in diakoptics of objects of a mathematical nature and the accompanying computational difficulties. This is the network approach in diakoptics. It should be emphasized that the designation of the external characteristics in the form (ab,c) in formula (7) are now considered as indicators of the method in which the NECS is connected.

Formula (7) cannot be used to analyse networks with controlled sources, unlike formula (4), since the order of the node numbers (the orientation of the NECS) in the designation of the external characteristics is important. The generalization of formula (7) necessitates matching the characteristics of the subnetworks by choosing one of the external nodes as a basis node. To fix our ideas, the number of the basis node is indicated as the first number in the pair of numbers before the comma. Let this node be node c. Here the last term of formula (7) must be represented in the form of two terms, taking into account the identity [5]:

$$\Delta(a,b) = \Delta(ca,b) + \Delta(cb,a). \tag{8}$$

This implies the generalized formula:

$$\Delta = \Delta_1(a,b,c)\Delta_2 + \Delta_1\Delta_2(a,b,c) + \Delta_1(cb,a)\Delta_2(b,c) + \Delta_1(ca,b)\Delta_2(a,c) + \\ + \Delta_1(ab,c)\Delta_2(ca,b) + \Delta_1(ba,c)\Delta_2(cb,a). \tag{9}$$

Since the notation in the formula obtained of the form (ab,c), as in (7) also assumes that there are no path 2-trees, but the connection of the NECS, formula (9) is found to be more general than similar formula for the bisection of a unistor graph [16]. Here, the subnetwork can contain not only y-branches and VCCS, but also components of any other type. Note that the formulae of the bisection of a unistor graph with respect to two or more nodes become considerably more complicated if its basis node is not one of the external nodes [15]. A network approach always enables one to choose one of the external subnetwork nodes as the basis node. The main advantage of this approach is that it excludes the occurrence of duplications when obtaining the external characteristics of a subnetwork which is inevitable in other approaches.

Finding the determinants of the derived networks containing additional NECS is a more complicated problem than finding the determinants of networks constructed by connecting two or three nodes of a subnetwork. This is important when a SNF is constructed without using a computer. The number of derived networks containing additional NECS can be reduced from six to four by modifying formula (9), taking into account identity (8) [17]. The formula derived in this way is redundant at the level of combining the external characteristics of subnetworks and takes the form

$$\Delta = \Delta_1(a,b,c)\Delta_2 + \Delta_1\Delta_2(a,b,c) + \Delta_1(a,c)\Delta_2(b,c) - \\ - \Delta_1(ab,c)\Delta_2(ba,c) + \Delta_1(b,c)\Delta_2(a,c) - \Delta_1(ba,c)\Delta_2(ab,c). \tag{10}$$

Hierarchical bisection of the network provides for repeated bisection of each subnetwork using formulae (4), (7), (9) or (10). The only constraint when using these formulae is the location of like generators and receivers in different subnetworks. The cases considered below of determining the parameters of individual components and subnetworks are very clearly shown in the figure of the circuit. Taking into account topological degeneration considerably simplifies the circuit analysis and reduces the possibility of errors occurring when the SNF is set up without using a computer.

Special cases of parameter determination. Such cases correspond to the following topological degeneracies: 1) a suspended y-branch and z-loop, 2) a degenerate VCVS and CCCS, 3) suspended VCCS, VCVS and CCCS, 4) a VCCS, the generator and receiver of which form a section, and 5) a pair of VCCS, the unlike generator and receiver of which form a section. In order to reduce the number of operations when the SNF is set up, the above special features of the network must be taken into account before the parameters are found from the general formulae, i.e. after checking the network for degeneracy and simplifying it.

Special cases of formulae (2) and (3) for $\Delta^y = \Delta_z = 0$ correspond to extraction of a suspended *y*-branch and *z*-loop. As in the case of a VCCS, VCVS and CCCS are said to be degenerate if their generator and receiver are connected in parallel. These VCVS and CCCS with parameters k and β are best considered as a subnetwork and isolated using (4), i.e.

$$\Delta = (1 \pm k)\,\Delta_2(a, b) \qquad (11)$$

and

$$\Delta = (1 \pm \beta)\,\Delta_2(a, b)\,, \qquad (12)$$

where a and b are external nodes of the subnetwork.

Here degenerate controlled sources are assumed as the first subnetwork. The signs of the parameters of the controlled sources are positive (negative) when the orientations of the generator and receiver are the same (opposite). This follows from the generalized unistor graph method and Fig. 1.

A voltage controlled current source is said to be suspended, when on one of the nodes of the network, apart from the generator and receiver of this VCCS, only current generators (Fig. 3, on the left) or only voltage receivers (Fig. 4, on the left) are incident. The position of a suspended VCCS in the network is similar to the position of a suspended *y*-branch, and to extract the parameter s of the suspended VCCS the following formula is used

$$\Delta = \pm\, s\,\Delta_s\,, \qquad (13)$$

where Δ_s is the determinant of the network, formed from the network with suspended VCCS by eliminating its receiver and contracting its generator (see Fig. 3, on the right) or, conversely, by eliminating its generator and contracting its receiver (see Fig. 4, on the right). A positive (negative) sign on the parameter s corresponds to similar (opposite) orientations of the generator and receiver of the suspended VCCS with respect to the node considered. Note, that in Figs 3 and 4 the cases of positive and negative signs respectively are shown.

Formula (13) and the corresponding transformations (see Figs 3 and 4) immediately follow from formula (9). If a suspended VCCS is assumed as the first subnetwork, then all terms of this formula will be equal to zero

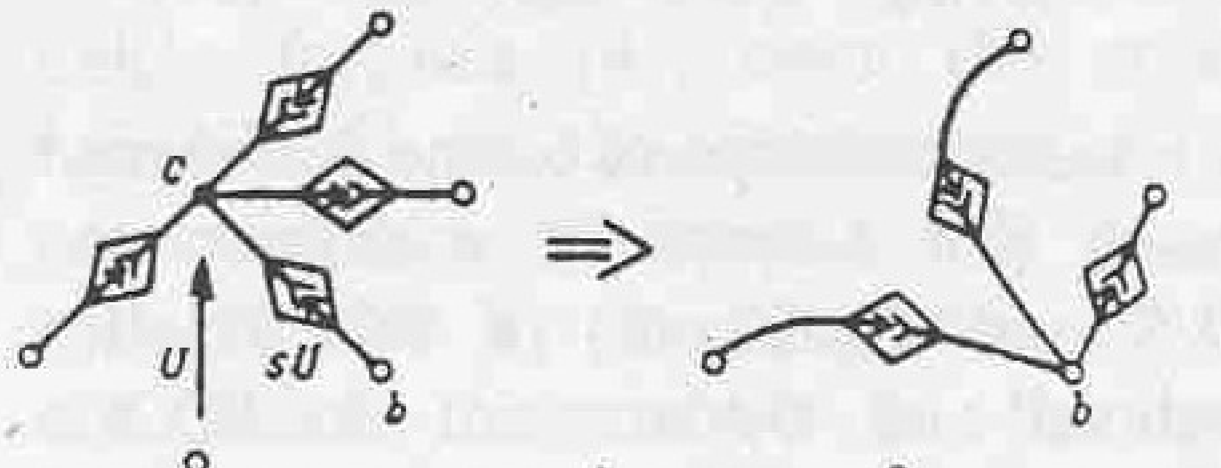

FIG.3

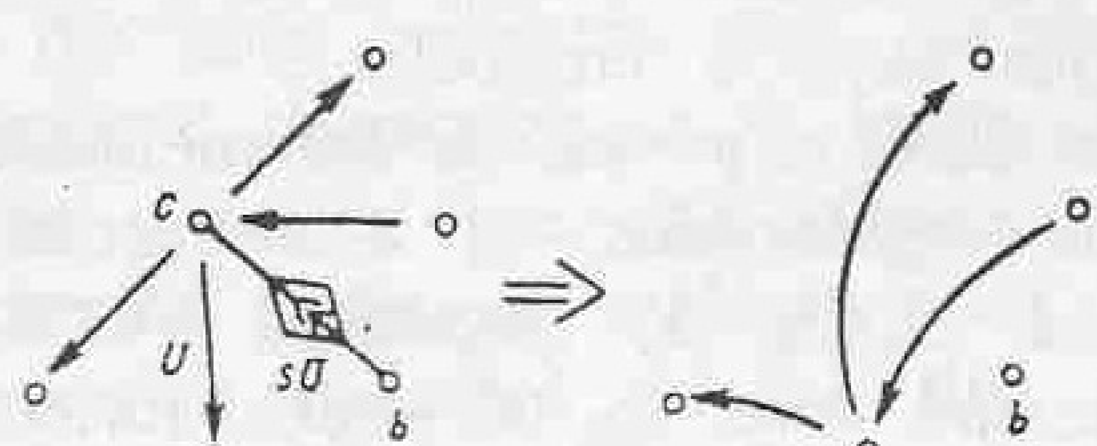

FIG.4

for the fourth term (For Fig. 3) or except for the fourth and sixth terms (for Fig. 4). The presence of two terms in the last case implies that formula (9) may contain duplications in a hidden form, i.e. at the level of a combination of the external characteristics of the subcircuits. To ensure that there are no duplications, it is necessary to choose a subcircuit containing no controlled source, as one of the subcircuits.

A voltage controlled source is said to be suspended, when on one of the nodes of the network, apart from the generator and receiver of this VCVS, only current generators are incident, as shown in Fig. 5, on the left. The determinant of the network containing suspended VCVS with parameter k is found from formula

$$\Delta = (1 \pm k)\,\Delta^{k}\,, \qquad (14)$$

where Δ^k is the determinant of the network, formed from the initial network by eliminating the suspended VCVS (see Fig. 5, on the right). Formula (9) is also used to prove formula (14). Of the terms of this formula only the fourth will be significant.

A current controlled source is said to be suspended, when on one of the nodes of this network, apart from the generator and receiver of this CCCS, only voltage receivers are incident, as shown in Fig. 6, on the right.

To extract the parameter β of the suspended CCCS the following formula is used

$$\Delta = (1 \pm \beta)\,\Delta^{\beta}\,, \qquad (15)$$

where Δ^{β} is the determinant of the network formed from the initial network by eliminating the suspended CCCS (see Fig. 6, on the right).

The third, fourth and sixth terms are taken from formula (9) for this case. The signs of the parameters k and β in formulae (14) and (15) are chosen to be similar to the sign of the parameter s in formula (13). The cases of positive and negative signs respectively are shown in Figs 5 and 6.

NECS generators (NECS receivers) are allowed among current generators (voltage receivers) in Figs 3—6. This is explained by the fact that the generators and receivers of the NECS, like current generators and voltage receivers, cannot lead to combination of the nodes in the network. Notice also, that a suspended VCCS is, in essence, a NECS with parameter s. Therefore formula (13) can

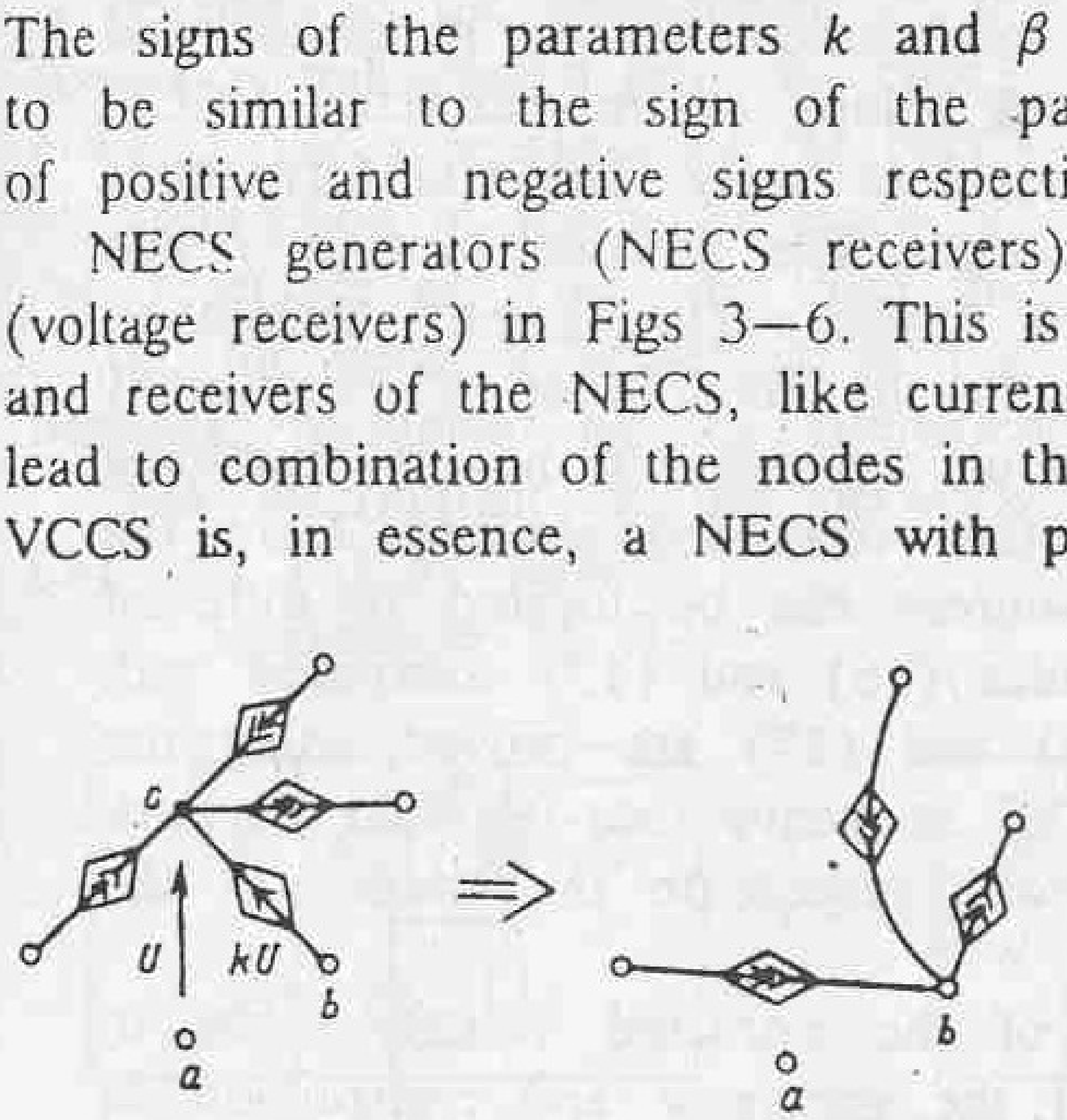

FIG. 5

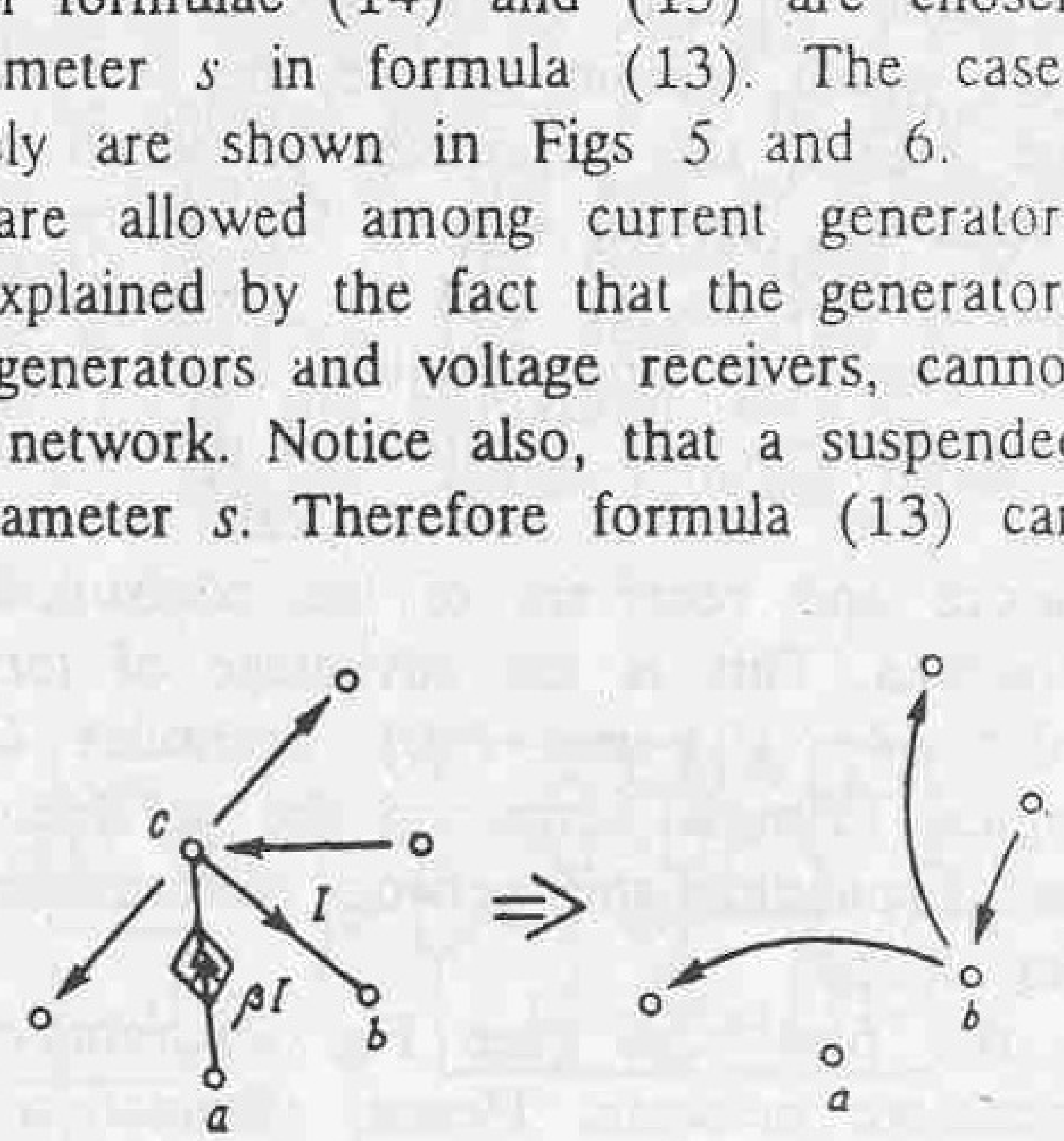

FIG. 6

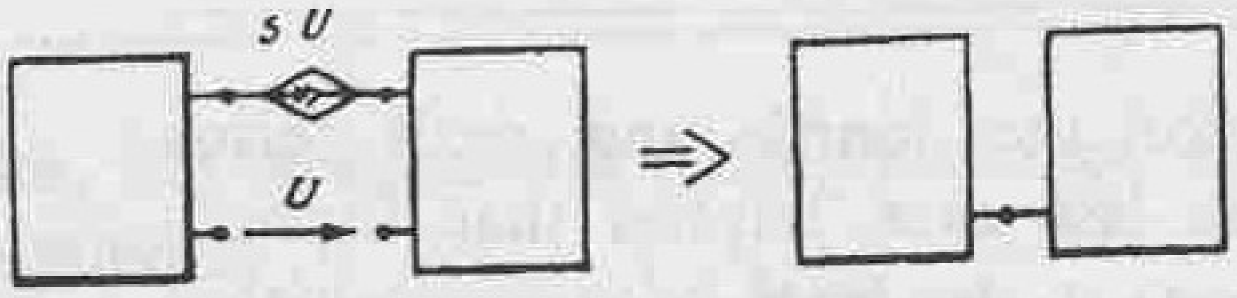

FIG.7

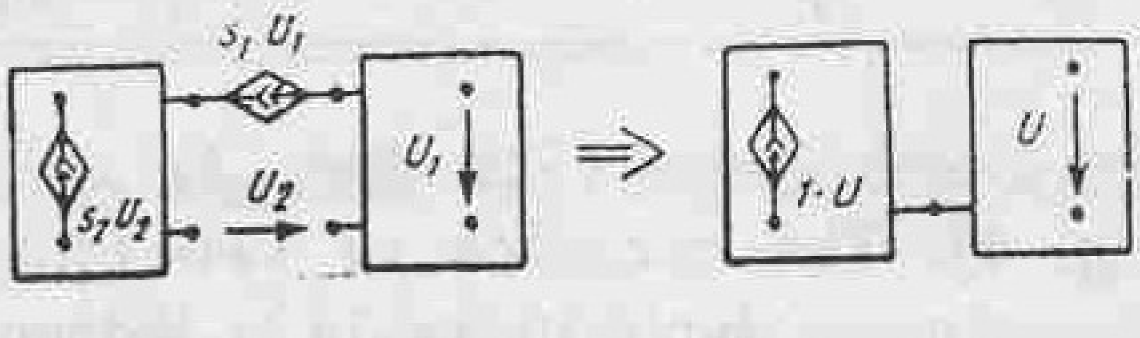

FIG.8

be used to extract the parameter of the NECS, if its arrangement in the network is the same as the arrangement of the suspended VCCS (see Figs 3 and 4).

Two other cases of extraction of the parameters of the VCCS, which are not a consequence of diakoptic formulae, are also of great importance. In both cases a section of the network, formed by the generator and receiver, is considered. In the first case the generator and receiver belong to the same VCCS, and in the second case to different VCCS, The first case is shown in Fig. 7, on the left, and the second one in Fig. 8. In the first case parameter extraction is carried out in accordance with formula

$$\Delta = \pm s \Delta' , \tag{16}$$

where Δ' is the determinant of the network, formed from the initial network by eliminating one and contracting the other branch of a section (see Fig. 7, on the right). The positive (negative) sign on the parameter s corresponds to the same (opposite) orientation of the generator and receiver with respect to the section.

In the second case the network determinant is found from the formula

$$\Delta = \pm s_1 s_2 \Delta'' , \tag{17}$$

where Δ'' is the determinant of the network formed from the initial network as a result of the following transformations: 1) elimination of one and contraction of the other branch of the section and 2) replacement of the generator and receiver, which remain outside the section and belong to the extracted VCCS, by one NECS, the parameter of which is equal to unity. The transformations mentioned are carried out in Fig. 8, on the right. The positive (negative) sign of the parameters s_1 and s_2 correspond to opposite (the same) orientation of the generator and receiver with respect to the section. It is noteworthy that the subnetworks, located on the left and on the right of the corresponding sections (see Figs 7 and 8) can contain other controlled sources. Here the generators and receivers of the controlled sources can be located in different subnetworks. This is the advantage of formulae (16) and (17) compared with formulae (4), (9) and (10). Formulae (16) and (17) are proved using the topological formula extracting the parameter of an active four-terminal network [9] and expanding the network determinant with respect to the circuits of the transfer loops.

In the first case (see Fig. 7) removal of the extracted VCCS results in a degenerate network. Hence, elimination of the generator and contraction of the receiver, on the one hand, will not result in loss of connectivity of the network, and on the other hand, will not prevent the formation of the transfer loops by the generators and receivers of other controlled sources.

In the second case (see Fig. 8) not one of the extracted VCCS can be eliminated from the network, since it will result in its degeneracy. Thus, their generators and receivers are necessarily involved in the formation of all circuits of the transfer loops of the network. Here each circuit is produced by the generator $s_1 U_1$, which excites the receiver U_2 via the left and right subnetworks, and the generator $s_2 U_2$ extends the chain of the transfer loops, which are closed in the receiver U_1. Consequently, the proof of the transformation in Fig. 8 can be reduced to proving the transformation in Fig. 7, by interchanging the position of the receivers U_1 and U_2 and then assigning to VCCS the status of the NECS. Such a network change will be correct if an inverse rule of sign determination is used compared with formula (16). Here one takes into account the fact that the section of the network contains unlike generator and receiver, i.e. rearrangement occurs [12].

Since the extracted VCCS in Figs 7 and 8 cannot be eliminated because of network degeneracy, formulae (16) and (17) can be applied in relation to the parameters of the NECS. Notice also, that if the subnetworks (see Figs 7 and 8) are independent of one another, it is convenient to expand the network determinants using Feussner's formula [3, 5]

$$\Delta = \Delta_1 \Delta_2 .$$

This formula represents the simplest case of network bisection.

An example of analysis of an electronic network. We will consider the equivalent circuit of an operational converter [11], shown in Fig. 9,*a*.

The voltage transfer SNF is derived from formula (1). Here the numerator is obtained as the determinant of the initial network, in which the independent source is transformed into the NECS, which is controlled by the output voltage $E = 1 \cdot U$. The appearance of the NECS in the network enables it to be simplified by contraction of the susceptance pC and resistance r_2, and also by eliminating the reactance pL (see the third and fourth forms of the connections). As a result, the network presented in Fig. 9,*b* is obtained. The determinant of this network, multiplied by the coefficient $p^2 CL$, is the numerator desired.

The determinant of the network in Fig. 9,*b* is expanded using formulae (3) and (6) as follows.

1. Elimination of R_2.

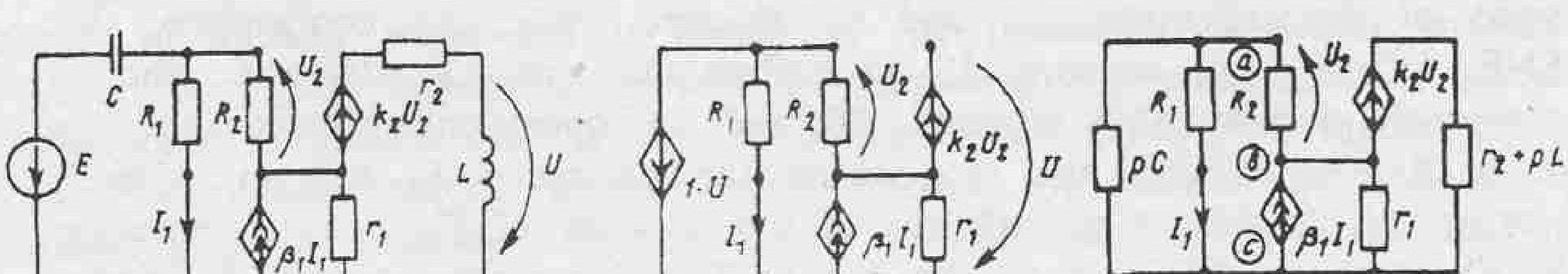

FIG 9

1.1. Transformation of $k_2 U_2$ into the NECS. Use formula (17) for $1 \cdot U_2$ and U.

1.1.1. Transformation of $\beta_1 I_1$ into the NECS. Elimination of r_1. Use formula (17) for $1 \cdot I_1$ and U. Contraction of R_1. Derivation of the degenerate NECS with parameter equal to unity, having the same orientation as the generator and receiver ($\Delta = 1$).

1.1.2. Elimination of $\beta_1 I_1$. Contraction of r_1. Elimination of R_1. Derivation of the degenerate NECS with parameter equal to unity, having an orientation opposite to that of the generator and receiver ($\Delta = -1$).

1.2. Elimination of $k_2 U_2$. Elimination of r_1. Use formula (17) for $\beta_1 I_1$ and U. Contraction of R_1. Derivation of the degenerate NECS ($\Delta = 1$).

2. Contraction of R_2. Elimination of $k_2 U_2$, $\beta_1 I_1$ and r_1. Elimination of R_1. Derivation of the degenerate NECS ($\Delta = 1$).

Hence

$$\Delta_N = p^2 CL \left\{ R_2 \left[k_2 (\beta_1 r_1 - R_1) + r_1 \beta_1 \right] + r_1 R_1 \right\}.$$

The denominator of the SNF is found as the determinant of the network formed from the network in Fig. 9,*a* as a result of contraction of the branch E and elimination of the branch U. The network derived is shown in Fig. 9,*b*. We use formula (9) to expand the determinant of this network. The network is separated into two subnetworks with respect to the nodes a, b and c. The left-hand subnetwork is assumed to be the first (second) in order. In accordance with the half division principle [17] the component R_2 and the generator $\beta_1 I_1$ are assigned to the first subnetwork, and the component r_1 to the second. Taking account of the factors, which are equal to zero, significant terms in formula (9) are the first, third and fourth. Hence, we obtain

$$\Delta_D = R_1 (r_1 + r_2 + pL) - \beta_1 r_1 (r_2 + pL) + (1 + pCR_1) \times$$
$$\times \left[(R_2 + r_1)(r_2 + pL) + R_2 r_1 (1 + k_2) \right].$$

As can be seen, the required SNF Δ_N / Δ_D is formed in compact form and contains no duplications. The use for this purpose of known methods, which enable all types of passive components and controlled sources to be taken into account [7, 11], involves a much greater volume of the intermediate operations and calculations. The SNF derived in [11] is equivalent to the one above with the signs of the parameters β_1 and k_2 changed. Note, for comparison, that the SNF derived here contains 15 multiplications and 13 additions, whereas the previously derived SNF required 25 and 15 operations respectively.

We shall now consider the case, when the parameter k_2 has an infinitely large value. To do this, it is sufficient in the network of Fig. 9,*a* to transform the VCVS into the NECS and take $k_2 = 1$. The appearance of NECS implies the elimination of the component R_2. Therefore, we shall not take into account its representation in Fig. 9,*b*.

The determinant of the modified network in Fig. 9,*b* is expanded using formula (6) as follows.

1. Use formula (17) for $1 \cdot U_2$ and U.

1.1. Transformation of $\beta_1 I_1$ into NECS. Elimination of r_1. Use formula (17) for $1 \cdot I_1$ and U. Contraction of R_1. Obtain the degenerate NECS ($\Delta = 1$).

1.2. Elimination of $\beta_1 I_1$. Contraction of r_1. Elimination of R_1. Obtain the degenerate NECS ($\Delta = 1$).

Hence,

$$\Delta_N = p^2 CL (\beta_1 r_1 - R_1) .$$

The modified network of Fig. 9,*c* is used to find the denominator of the SNF. There is a section in this network which is formed by the impedance $r_2 + pL$ and the NECS $1 \cdot U_2$. It requires simplification by contraction of $r_2 + pL$. This results in the elimination of $\beta_1 I_1$ and r_1. In the network obtained there is a suspended NECS $1 \cdot U_2$. The determinant of this network is expanded using formula (13) and is equal to $1 + pCR_1$ (the parameter of the suspended NECS is taken with positive sign). Hence, we have

$$\Delta_D = r_1 (1 + pCR_1) .$$

The derived SNF Δ_N / Δ_D is equivalent to the SNF obtained above, provided that $k_2 = \infty$. The advantage of the proposed method of taking into account infinitely large parameters is in that before analysing the network it is simplified using physical considerations. In the case considered these simplifications are the transformation of the VCVS into the NECS and elimination of the component R_2.

CONCLUSIONS

A new network element, a non-eliminated controlled source, has been investigated. This is used to determine the numerators of the transfer SNF, to extract the parameters of the controlled sources and subnetworks and to analyse networks with ideal controlled sources.

A direct circuit solution of the problem of the symbolic analysis of a network with a controlled source has been found, which generalizes similar Feussner solution for a passive network. Unlike the network approach of Barrows and Hoang, the solution proposed does not require laborious enumeration of the chains of transfer loops and is based on the parameter extraction method.

The topological method developed for extracting the parameters of passive components, controlled sources and subnetworks, compared with matrix, graph, and set-theoretic methods, is a universal method. It enables the SNF to be obtained without mutually cancelled terms, on the basis of the circuit diagram and the networks derived from it. The use of the method does not involve any abstract concepts (graph, matrix etc.). As a result, there no break with physical concepts.

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