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Parameters extraction technique for optimal network functions of SC circuits

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Abstract — **The parameter extraction technique for symbolic analysis of switched capacitors circuits have been presented. The proposed extraction formulae reduce of cancellation sum-of-product terms amount in transfer function expression. For equivalent circuits of SC the models based on voltage controlled current sources with capacities parameters have been used to avoid the cancellations appearance at the circuit level. The proposed technique is implemented in symbolic circuit analysis program Cirsym. Illustrative examples on SC circuit symbolic analysis are given.**

Key words: switched-capacitor, symbolic analysis, circuit determinant, nullor, n-poles extraction.

I. Introduction

Time-discrete circuits containing switched-capacitors (SC) are widely used in mixed-signal systems, such as electronic filters, Sample-Hold circuits, peak detectors, mixers, modulators of shift-keying techniques, analog-digital or digital-analog converters [1, 2].

The Spice analysis of switched circuits is very laborious and time consuming. It consists in the steady state transient analysis with the input signal swept by a harmonic signal of a given frequency, the selection of an AC component of output signal, and its comparison with the input. This procedure must be repeated for more frequencies.

Symbolic analysis in contrast to numeric calculation techniques provides the way to get analytical solution for transfer function computation, for determination of required network mode conditions, for fault modeling and testing of SC circuits [3-12]. The symbolic techniques are employing the ideal models of circuit elements: capacitors, ideal switches (short-circuited connection is corresponding for on-state, open circuit is corresponding for off-state), controlled sources and ideal operational amplifiers. The network equations derived from the SC equivalent circuit are the *z*-domain charge equations, where *z* is the operator of the *z*-transform [13].

For symbolic analysis of the SC circuit the methods of nodal charge equations using matrix calculus are usually applied. Some of them are based on nodal analysis, such as technique proposed in [10], the others have deal with the modified nodal matrix [5, 6]. But matrix model of SC circuit consist of the equal summands with opposite sign. It leads to appearing of the cancellation sum-of-product terms.

The topological methods are implementing graph theory for symbolic circuit analysis. The techniques based on the signal flow graph [4, 7, 8], Mason-Coates graph [3, 11], and Nathans graph [9] have been developed. The main drawback of topological approach is a time-consuming combinatorial search of branches and loops. The appearing of cancellation terms is also unavoidable for topological methods.

In this paper we propose the SC circuit symbolic calculation technique based on the generalized parameter extraction method (GPEM) which is cancellation-free [14-18]. In Section II we explain the basic idea of SC circuit analysis based on network determinant expansion. The comparison of the network function evaluation by means of proposed technique and matrix-based approach are presented in Sections III and IV.

II. Description of the Method

A. The basics of generalized parameter extraction method

Let's express the transfer function of electronic network as

$$H = N / D, \tag{1}$$

where N is the determinant of the network, in which the independent source and output response are replaced by nullor, and D is the determinant of the network, in which the independent excitation and the output response are zero.

For network determinants calculation the parameter extraction formula was presented in [14]:

$$\Delta = \chi\, \Delta(\chi\to\infty) + \Delta(\chi = 0), \tag{2}$$

where χ is a parameter of linear circuit element, $\Delta(\chi\to\infty)$ and $\Delta(\chi = 0)$ are the determinants of the circuits in which the parameter of extracted element $\chi\to\infty$ or $\chi = 0$ respectively.

For nullor extraction the special formula must be used:

$$\Delta = \pm\Delta_n, \tag{3}$$

where Δ_n is the determinant of the circuit after the nullor number n extraction.

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The procedure of nullor extraction can be formalized by the following steps [18]:

1. The choice of the supporting nodes. First of the supporting nodes should be connected to norator, and the second to nullator. If there is a common node of a nullator and a norator it must be chosen as single supporting node. Note that supporting node may correspond to the ground node.

2. The terminals of non-extracted norators and current sources connected to supporting node are moved to the opposite node of extracted norator. In that case the non-extracted nullators and controlling voltages keep connections to supporting node. Then in the same way, the terminals of non-extracted nullators and controlling voltages connected to supported node are moved to the opposite node of extracted nullator. In that case non-extracted norators and current sources keep connections to supporting node.

3. A norator and a nullator of the extracted nullor are deleted from circuit. In case of two supporting nodes they must combine.

4. If extracted norator and nullator have got the same orientation with respect to the supporting node then the determinant sign will be positive. Otherwise the sign will be negative. In case of two supporting nodes the inverted rule is needed.

Other special cases of circuit topology transformations provided by GPEM can be found in [14-18].

B. The GPEM-based technique for SC circuits symbolic analysis

For symbolic analysis of SC circuits in the z-domain the clock-free equivalent network of switched capacitor is needed. In the Table I the equivalent networks for typical cases of SC configurations are presented. We used here the SC model based on the usage of voltage controlled current source (VCCS) with capacities parameter [3] instead of common model with two-port complex capacities.

The symbolic expressions of switched circuits transfer functions are calculating by means of circuit-algebraic formulae as well as for analog networks [14]. Of course it is important to take into account that SC equivalent circuit consists of the out-of-phase input and output. As example in Table II we present the circuit-algebraic formulae for analysis of the three-terminal circuit with two-phase control. For that case four voltage transfer function variations different by phases of input and output voltage have been derived. In case of circuit controlled by more than two phases the amount of circuit-algebraic formulae will increase correspondingly.

The capacitance extraction formula easy developed from expression (2) in consequence of current and charge compatibility:

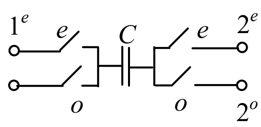
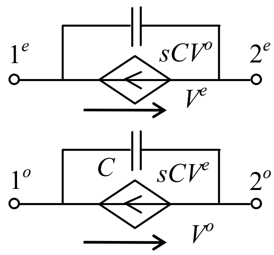
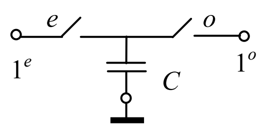
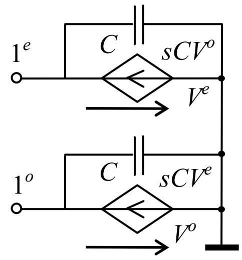
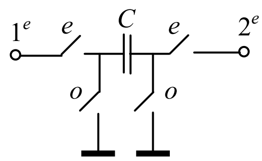
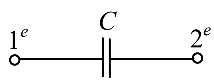
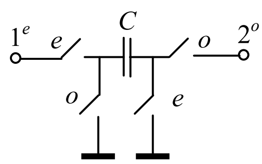
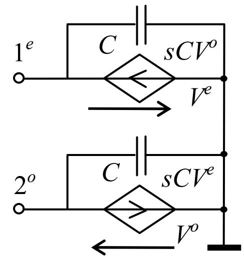
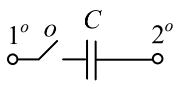
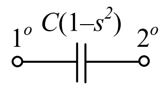
$$\Delta = C\,\Delta(C\to\infty) + \Delta(C = 0). \tag{4}$$

For extraction of z-capacitance just replacement of parameter C by C_z in expression (4) is needed:

$$\Delta = C_z\,\Delta(C_z \to\infty) + \Delta(C_z = 0). \tag{5}$$

For the controlled source parameter extraction the formula (2) must be used.

TABLE I. THE EQUIVALENT NETWORKS FOR TYPICAL CASES OF SC CONFIGURATIONS

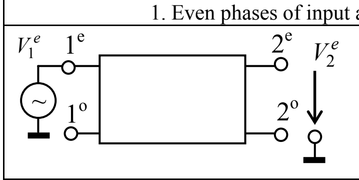
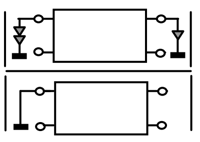
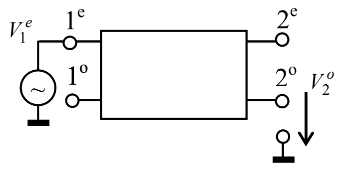
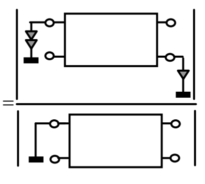
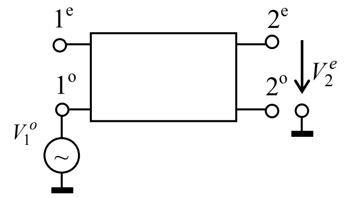
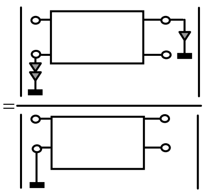
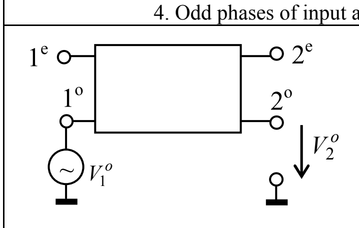
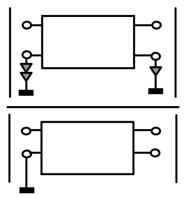
	SC topology	Complex equivalent circuit (operator $s = z^{(-1/2)}$)
1	Switched capacitor in general form	
		
2	Grounded SC for voltage transfer	
		
3	SC with single-channel voltage transfer	
		
4	Non-grounded SC for voltage transfer	
		
5	SC with single switch	
		

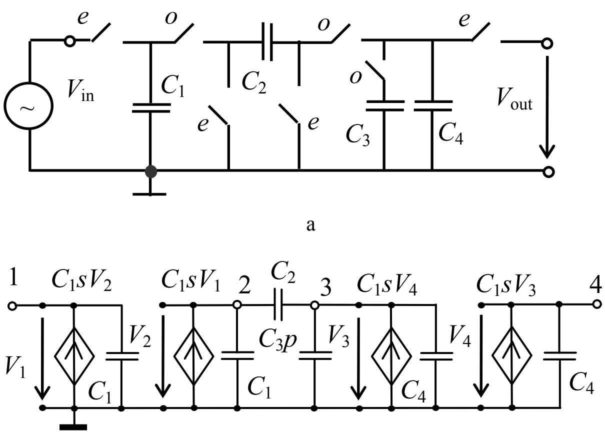
III. EXAMPLE OF THE SOLVING

A. Solving by GPEM-based technique

Let's consider the implementation of proposed technique to network function calculation of simple SC circuit presented in Fig. 1 (a). The z-equivalent circuit shown in Fig. 1 (b) where $s = 1 - z^{-1}$, $p = 1 - s^2$ is formed by means of Table I.

TABLE II. CIRCUIT-ALGEBRAIC FORMULAE FOR TRANSFER VOLTAGE FUNCTION OF THE THREE-TERMINAL CIRCUIT WITH TWO-PHASE CONTROL

Four-terminal networks model	**Circuit-algebraic formula**
1. Even phases of input and output voltages	
	 $H_{ee}=\frac{V_2^e}{V_1^e}=$
2. Even phase of input voltage and odd phase of output voltage	
	 $H_{oe}=\frac{V_2^o}{V_1^e}=$
3. Odd phase of input voltage and even phase of output voltage	
	 $H_{oe}=\frac{V_2^e}{V_1^o}=$
4. Odd phases of input and output voltages	
	 $H_{oe}=\frac{V_2^o}{V_1^o}=$

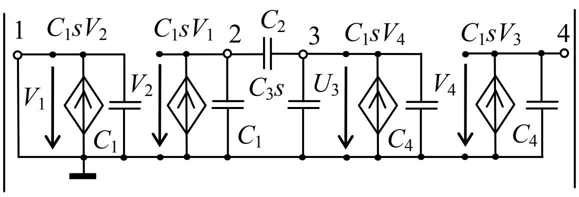


a

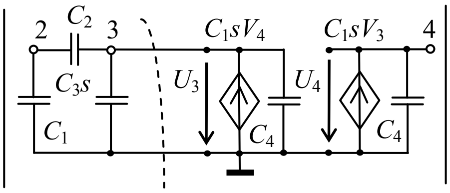
b

Fig. 1. The SC test-circuit (a) and *z*-equivalent circuit (b)

In accordance with (1) let's expressed the circuit for denominator of transfer function as following:

$\Delta_{11}=$  (6)

Now we transform expression (6) in consideration of voltage V_1 equal to zero:

$\Delta_{11}=$  (7)

For simplification of determinant calculation let`s divide the circuit in two subcircuits by the nodes 3 and 0 as shown in (7) by dashed line. And then we evaluate the circuit determinant by means of extraction formulae (2) and (3):

$$D=[C_1(C_2+C_3s)+C_2C_3s]C_4+(C_1+C_2)(-C_4^2s^2+C_4^2). \quad (8)$$

The circuit determinant of transfer function numerator is calculated in the same way:

$$N=C_1C_2C_4z^{(-1/2)}. \quad (9)$$

As can be seen there is no cancellation terms in transfer function $H = N / D$ evaluated by proposed technique.

B. Solving by Nodal Analysis Method

To compare the solution of a circuit with switched capacitors by the method presented above, we will calculate the same circuit by the usually used method of nodal charge equations using matrix calculus. The circuit in Fig. 1 (a) has four nodes, so the nodal matrix will be of the fourth.

$$\mathbf{C}_Z=\begin{array}{|c|c|c|c|}\hline C_1z^{-1/2} & -C_1z^{-1/2} & 0 & 0\\ \hline -C_1z^{-1/2} & C_1+C_2 & -C_2 & 0\\ \hline 0 & -C_2 & C_2+C_4+sC_3 & -C_4z^{-1/2}\\ \hline 0 & 0 & -C_4z^{-1/2} & C_4\\ \hline\end{array}$$

Transfer function can be expressed as $H = U_1/U_4 = \Delta_{14}/\Delta_{11}$. For calculation of matrix determinant let`s construct the binary decision diagram (BDD) as following [12]:

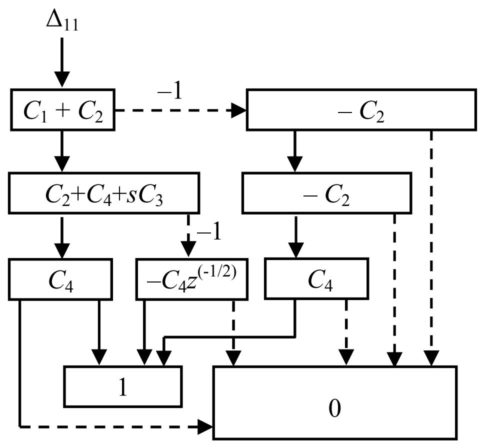


Fig. 2. Binary decision diagram for determinant Δ_{11} calculation

The result of BDD evaluation presented below:

$$\Delta_{11}=(C_1+C_2)[(\,C_2+C_4+sC_3)C_4-C_4^2z^{-1}\,)]-C_2^2C_4. \quad (10)$$

As can be seen there are two equal summands with opposite sign in the expression (10): $\pm\, C_2^2C_4$.

IV. Automatic Symbolic Analysis of SC Circuits

For automated analysis of large-scale SC circuits the computer program Cirsym developed by V. Filaretov can be used [14]. Cirsym is a symbolic analyzer a part of the software tool SCADS. The program can be downloaded from www.intersyn.net. The input data for program is SPICE-like circuit description. Note that in case of SC circuits the capacitor with parameter C and controlled source with capacities parameter must be presented as g- conductance and G- conductance correspondingly. Of course all parameter indexes are still the same. The operator $z^{(-1/2)}$ is considered as equivalent to complex operator s used in program. The symbolic expressions obtained by means of Cirsym needs to be transformed in z-domain by means of simple replacement of parameters g and G by C and $Cz^{(-1/2)}$ correspondingly.

Let`s consider the automated symbolic analysis of Fleischer-Laker biquad with two ideal OpAmp and twelve capacitors presented in Fig. 3. The equivalent circuit with VCCS-based SC models (see Table I) can be seen in Fig. 4. Note that controlling branches of controlled sources are not shown for purpose of presentation clearness.

We used the program Cirsym for calculation of transfer function for odd-phase of input and output voltages. The result expression in which the each capacities parameter is represented by corresponding index is listed below:

H_{cirsym}:= s*a*(s*(-s^2*d*h*b*(l+k)+d*(-s^2*e*(j+h)*k+ b*(-s^2*l*(k-l)+(l+k)*l+(l+j+h+k)*g)))+d*s*e*(j+h)*k)+ s^2*j*d^2*b*(-k-l)*(-s^2+1)+s^2*b*d^2*(j+h)*k* (s^2*-1)-s^2*d^2*b*(-s^2*(k*l+k^2)+(l+j+h+k)* (k+i))+d^2*b*(-s^2*k*(l+k)+(l+j+h+k)*(k+i)))
/
((l+k+j+h)*(s^2*a*d*b*(c+e*(1-s^2))
-s^2*b^2*d^2*(-s^2+1)-b*(f+b)*d^2*(s^2+1))); (11)

We can simplify the expression (11) by means of 22 cancellation terms reduction as following:

H := ((((k^2+(h+l+2*j)*k+l*j)*d-a*(l+h)*(l+k))*b- a*k*e*(j+h))*s^4+(((-2*k^2+(-2*h-2*l-i-3*j)*k+(-j-i)*l- i*(j+h))*d+a*((l+c)*k+l^2+l*c+c*(j+h)))*b+a*k*e*(j+h))*s^2 +d*b*(k+i)*(l+j+h+k))
/
((l+j+h+k)*b*((a*e-d*b)*s^4+(2*d*b+ d*f-a*(e+c))*s^2-(f+b)*d)). (12)

Let`s compare the expression (12) with determinant of Fleischer-Laker biquad circuit calculated by means of BDD-approach:

H_{BDD} :=((h+l+k+j)*(-g-l)*s^2*d*a*b+ (h+l+k+j)* s^2*l*d*a*b+(h+l+k+j)*(-i-k)*d^2*b-(h+l+k+j)*(-i- k)*s*d^2*b+(h+l+k+j)*s^2*k*d^2*b-(h+l+k+j)*s^2*k*d*e*a- (h+l+k+j)*s^3*k*d^2*b +(h+l+k+j)*s^4*k*d*e*a- s*(h+l)*s*(-l-k)*s^2*d*a*b+l*s^2*(-l-k)*d*a*b-s*(j+k)*s*(-l- k)*d^2*b+s*(j+k)*s^2*(-l-k)*d^2*b+k*s^2*(-l-k)*d^2*b- k*s^2*(-l-k)*d*e*a-k*s^3*(-l-k)*d^2*b+k*s^4*(-l-k)*d*e*a)
/
((h+l+k+j)*(-d^2*(-f-b)*b-d^2*s*b^2-d*s^2*a*e*b+ d*s^2*a*e*b+s*d^2*(-f-b)*b+s^2*d^2*b^2- s^2*d*a*(-c-e)*b-s^4*d*a*e*b)). (13)

As can be seen the amount of cancellation terms is increase to 32. The appearance of new 10 cancellation terms is a consequence of matrix calculus because switching capacitor matrix model include of some equal parameters.

In Table III we present the comparison of cancellation terms amounts in expressions obtained by means of GPEM-based technique and BDD method. We performed the calculation for VCCS-based equivalent circuit of Fleischer-Laker biquad and for circuit model based on two-ports.

TABLE III. Comparison of Terms Amounts in Transfer Function Expressions of Fleischer-Laker Biquad

	Circuit model	Calculation technique	Summands amount		Non-cancellation summands amount		Cancellation summands amount
			N	*D*	*N*	*D*	
1	VCCS-based circuit	GPEM	42	36	42	36	22
2		Matrix	66	44			32
3	circuit based on two-ports	GPEM	282	168			372
4		Matrix	420	360			702

It is possible to use the simplified SC models for symbolic expressions calculation of compact size without cancellations. But approximation may leads to significant lack of accuracy of network function. For example let`s consider the network function of Fleischer-Laker biquad circuit presented in [5]:

$H_{simplified}$:=((d*k+d*j-a*l-a*h)*s^4+(-2*d*k+a*l+a*g-d*j- d*i)*s^2+d*(k+i))/((a*e-d*b)*s^4+(-a*e+2*d*b- a*c+d*f)*s^2-d*(f+b)). (14)

This function is exactly corresponding to single-channel equivalent circuit shown in Fig. 5.

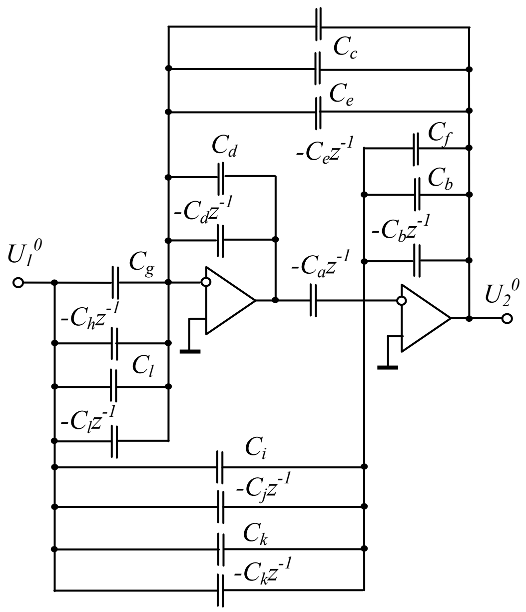


Fig. 5. Single-channel equivalent circuit of Fleischer-Laker biquad

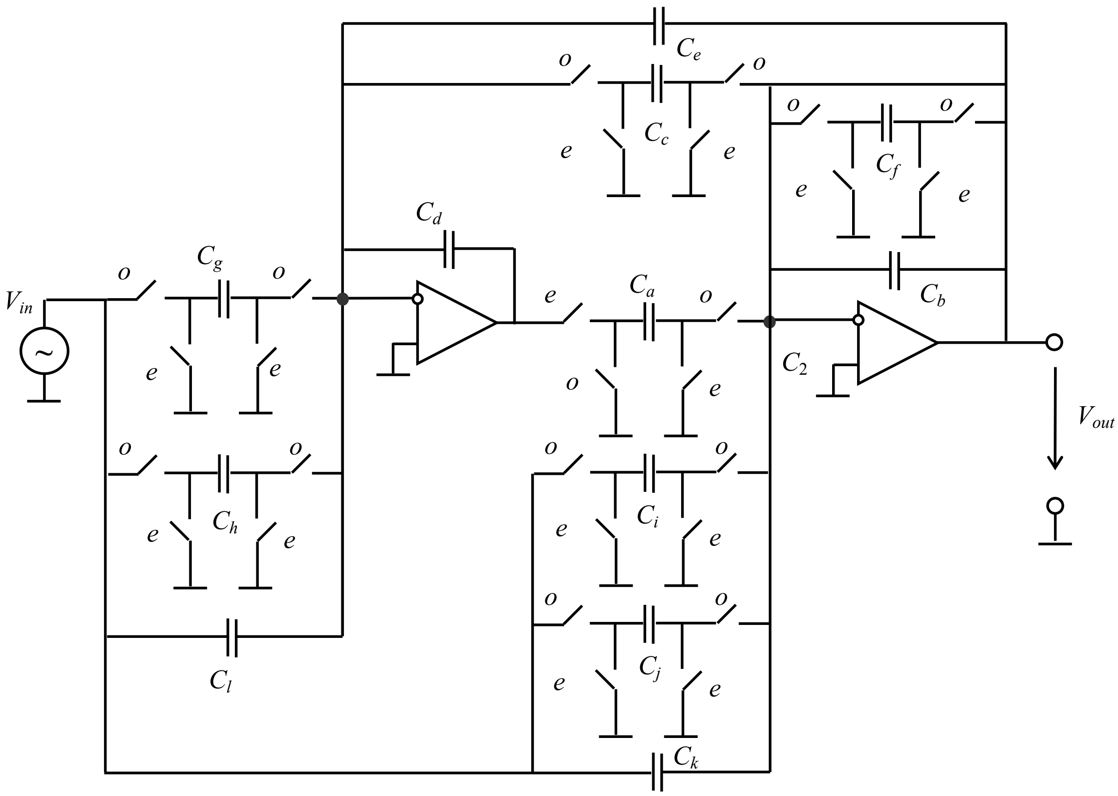


Fig. 3. Switched-capacitor biquad of Laker and Fleischer

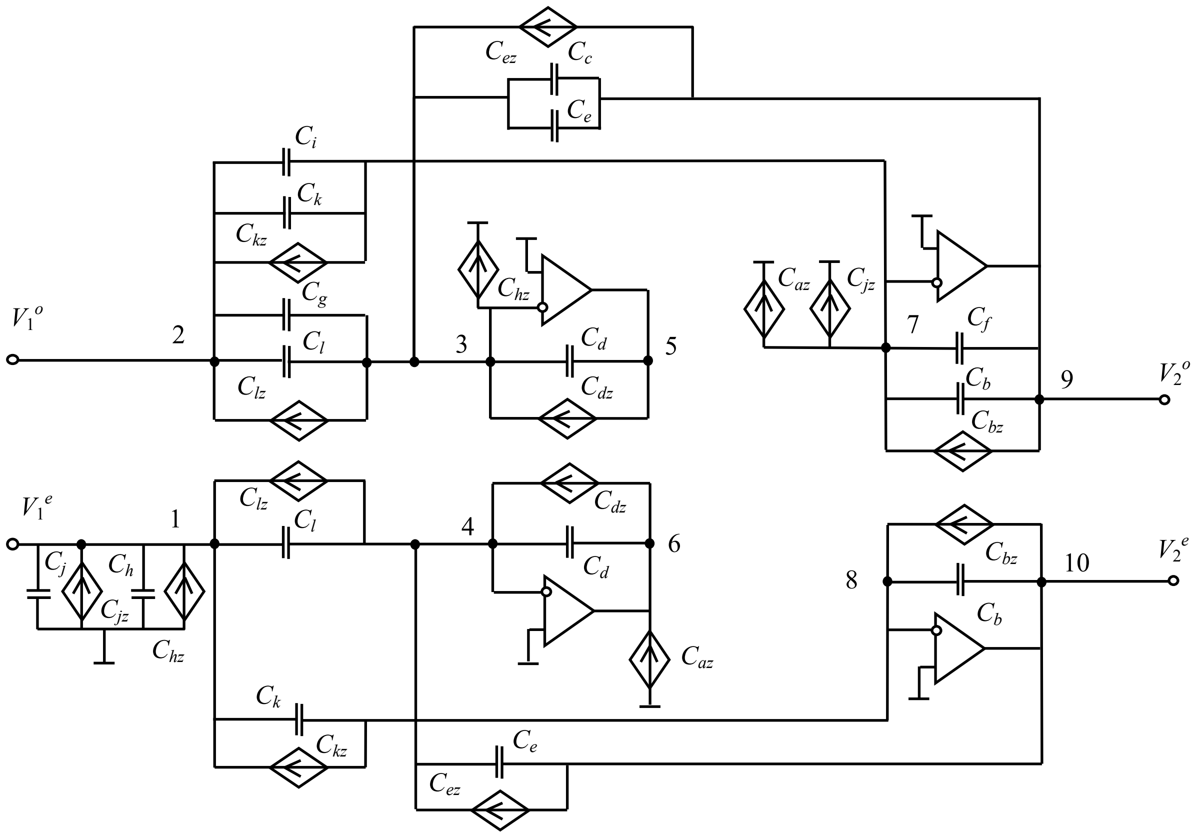
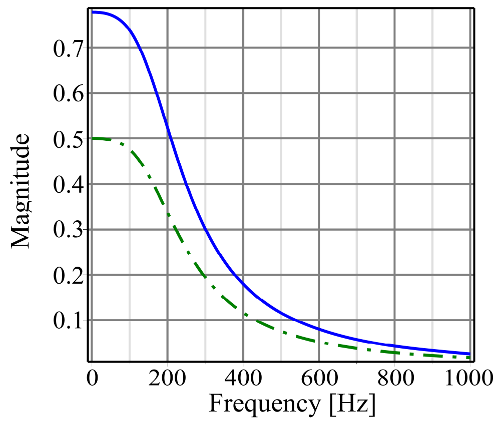
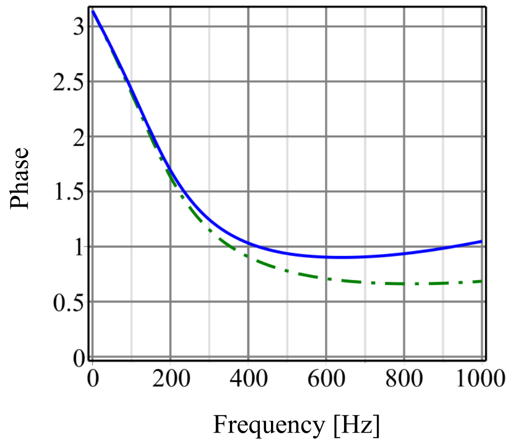


Fig. 4. VCCS-based equivalent circuit of Fleischer-Laker biquad

We performed the comparison of accuracy between expressions (12) and (14) for clock frequency of 10 kHz. Using Maple 18 software simulation results versus frequency *f* are shown in Fig. 6. The values of parameters elements are: C_a=276 *pF*; C_b=276 *pF*; C_c= 4 *pF*; C_d = 276 *pF*; C_e = 45 *pF*; C_f=2 *pF*; C_g = 4 *pF*; C_h = 2 *pF*; C_i = 1 *pF*; C_j = 3 *pF*; C_k = C_l = 2 *pF*. In that case the biquad circuit operates like low-pass filter with cutoff frequency of 190 Hz.



a



b

Fig. 6. Simulation of Fleischer-Laker biquad transfer functions (12) obtained by means of GPEM-based technique (blue line) and (14) presented in [5] (green dash dot line) versus frequency *f*

The result of transfer function (12) simulation is in good agreement with the numerical response computed by SPICE. But the amplitude-frequency characteristic error of the simplified equivalent circuit presented in [5] is more than 30%.

V. Conclusions

The technique for SC circuits symbolic analysis based on generalized parameter extraction method have been proposed. The GPEM approach does not require the matrix formation, which include a lot of the equal summands with opposite sign. The further reduce of cancellation provided by usage of VCCS-based equivalent circuit instead of common SC model with two-port complex capacities. The process of calculation symbolic network functions of SC circuits is automated by computer program Cirsym.

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